DOCUMENT RESUME

ED 070 662 24 SE 015 471

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TITLE The Performance of First Grade Students on a Nonstandard Set of Measurement Tasks.
INSTITUTION Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.
SPONS AGENCY National Center for Educational Research and Development (DHEW/OE), Washington, D.C.
REPORT NO WURDCCL-TR-211
BUREAU NO BR-5-0216
PUB DATE Dec 71
CONTRACT OEC-5-10-154
NOTE 25p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS *Cognitive Development; Conservation (Concept); *Elementary School Mathematics; Geometric Concepts; Learning Theories; Mathematics Education; *Measurement; *Research

ABSTRACT

First-grade students' understanding of several basic measurement processes was examined. Twenty randomly selected students from five first-grade classes in two different schools were individually examined on 11 measurement tasks. Seven of the tasks were adapted from items reported in the Gal'perin and Georgiev study, designed to test understanding of the unit of measure; four additional tasks required students to respond strictly on the basis of numerical clues. Students had difficulty with the tasks, which seemed to be the result of inability to conserve, inexperience with the specific measurement operations, and an ambiguity in several of the items. Findings showed that students did not have a stable concept of measurement and were not able to appreciate the value of a constant unit of measure, that there was little transfer between tasks, that tasks involving unequal quantities were easier than similar tasks involving equal quantities, and that numerical cues were almost as strong as physical cues in certain conservation tasks. (Author/DT)

TECHNICAL REPORT NO. 211

THE PERFORMANCE OF FIRST GRADE STUDENTS ON A NONSTANDARD SET OF MEASUREMENT TASKS

REPORT FROM THE PROJECT ON INDIVIDUALLY GUIDED ELEMENTARY MATHEMATICS PHASE 2: ANALYSIS OF MATHEMATICS INSTRUCTION

WISCONSIN RESEARCH AND DEVELOPMENT

CENTER FOR COGNITIVE LEARNING

RESEARCH COGNITIVE LEARNING DEVELOPMENT

Technical Report No. 211

THE PERFORMANCE OF FIRST GRADE STUDENTS ON A NONSTANDARD SET OF MEASUREMENT TASKS

By Thomas P. Carpenter

Report from the Project on
Individually Guided Elementary Mathematics
Phase 2: Analysis of Mathematics Instruction

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December 1971

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the United States Office of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the Office of Education and no official endorsement by the Office of Education should be inferred.

Center No. C-03 / Contract OE 5-10-154

Statement of Focus

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish a rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6, providing not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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Abstract

This study was designed to examine first-grade students' understanding of several basis measurement processes. Twenty Ss randomly selected from five first-grade classes in two different schools were individually examined on eleven measurement tasks. Seven of the tasks were adapted from items reported in the Gal'perin and L. S. Georgiev study, designed to test young children's understanding of the unit of measure. In these tasks, Ss were asked to measure and compare piles of rice using different-sized spoons, measure out lengths of string, etc. Four additional tasks were patterned after the Soviet problems except that Ss had to respond strictly on the basis of numerical cues with no physical evidence present.

Ss generally had difficulty with the tasks. Only one of the Soviet problems was answered correctly by more than half of the Ss tested. However, these difficulties did not appear to stem from an incorrect characterization of the unit of measure as hypothesized by Gal'perin and Georgiev. They seemed to be more the result of Ss' inability to conserve, their inexperience with the specific measurement operations, and an ambiguity in several of the items.

More specifically, the study showed that Ss did not have a stable concept of measurement, nor were they able to appreciate the value of a constant unit of measure. There was little transfer between measurement tasks. Measurement tasks involving unequal quantities were easier than similar tasks involving equal quantities. However, Ss readily shifted [illegible] of comparison of quantities, even on the same task. And finally, the study showed that numerical cues were almost as strong as physical cues in certain conservation tasks.

I
Introduction and Background

Measurement processes are currently becoming an established part of the mathematics curriculum of the early primary grades. In a survey of 39 completed or partially completed elementary mathematics series, Paige and Jennings (1967) found that by the first grade about half of the texts had introduced linear and liquid measurement.

Some current proposals advocate an even more extensive treatment of measurement concepts. The Cambridge Conference on School Mathematics (1963) and the K-13 Geometry Committee (1969) have advocated teaching measurement of length, area, and volume using both arbitrary and standard units in the early primary grades.

The mathematics program Developing Mathematical Processes (DMP), being developed at the Wisconsin Research and Development Center for Cognitive Learning, has made measurement processes the basis for developing fundamental number concepts (Romberg, Fletcher, & Scott, 1968). By grade 1 the arithmetic units include activities in which children choose arbitrary units of length and weight and use them to measure specified objects (Romberg & Harvey, 1969, 1970).

The proposed geometry unit for grades K-2 is to include measuring and comparing length, area, volume, weight, and time using both arbitrary and standard units of measure (Harvey, Meyer, Romberg, & Fletcher, 1969).

One of the dangers of introducing measurement concepts in the early grades is that children may develop a superficial concept of measure with which they are able to perform by rote the given operations, but are unable to assimilate the mathematical principles underlying the operations of measurement as a process. Research by Piaget and others (Flavell, 1963) indicates that young children are unable to compare physical quantities that have undergone various transformations. Since it is generally necessary to perform transformations on the quantity being measured or on the unit of measure, this research seems to have serious implications for teaching measurement processes in the early grades.

The work of two Soviet researchers, P. Ya. Gal'perin and L. S. Georgiev (1969), supports the contention that young children have difficulty with some of the basic concepts of measurement. From a series of fourteen measurement tasks given to a group of bright Soviet kindergartners, they concluded that these children taught by traditional methods lacked a basic understanding of the concept of a unit of measure.

Statement of Purpose

The purpose of this study was: (1) to give seven of the fourteen Gal'perin/Georgiev tasks* to American first graders to determine whether their responses matched those of Soviet students; and (2) to give a set of related tasks from which the dominant visual clues had been removed in order to better understand the nature of Ss' responses.

Mathematical Background

In order to understand how the Gal'perin and Georgiev studies relate to the process of measurement and to other measurement research,

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*The seven tasks are those contained in a series of articles by Gal'perin and Georgiev, translated in Kilpatrick & Wirszup, Soviet studies in the psychology of learning and teaching mathematics, 1969. The entire set of tasks is not available outside the Soviet Union.

specifically that of Jean Piaget, it is necessary to consider exactly what constitutes the measurement process.

Measurement can be defined as "the assignment of particular mathematical characteristics to conceptual entities in such a way as to permit (1) an unambiguous mathematical description of every situation involving the entity and (2) the arrangement of all occurrences of it in a quasi-serial order." (Caws, 1962) Mathematically, the process of measurement can be discussed in terms of functions mapping the elements of a domain into some mathematical structure (usually a subset of the real numbers) in such a way as to preserve the essential characteristics of the domain. (For a more complete treatment of a functional approach to measurement and for the definitions of the mathematical terms used in this discussion, see Blakers, 1967.)

The first requirement for the establishment of a measurement function is to recognize a domain of elements, D, which possesses a given attribute. (The term elements is used loosely and can include such things as quantities of a liquid that can be partitioned in an infinite number of ways into distinct elements.) By empirical procedures the domain is given a structure, usually involving the establishment of operations and relations on the objects of the domain. In most common measurement functions this structure is imposed by first establishing a procedure for comparing elements of D on the basis of the given attribute and using this procedure to define an equivalence relation, $\sim$, on the elements of D. This equivalence relation is used to partition D into equivalence classes $\tilde{d}$, thereby creating a set $\tilde{D}$ of the equivalence classes of D. The procedure for comparing elements of D also allows one to define an order relation < on D, which turns out to be a strict total order relation on D. That is, for every two elements d_1 and d_2 of D, exactly one of the following holds: $d_1 = d_2$, $d_1 < d_2$, or $d_2 < d_1$. This order relation, $\leq$, yields a corresponding order relation, < on $\tilde{D}$, defined as follows:

given $\tilde{d}$ and $\tilde{e}$ in $\tilde{D}$, $\tilde{d} < \tilde{e}$ if $d_1 < e_1$ for any d_1 in $\tilde{d}$, e_1 in $\tilde{e}$.

Further, an operation which is both commutative and associative is defined on the set D and extended in a natural way to the set $\tilde{D}$. Thus, $(D, *, <)$ assumes the structure of an ordered abelian semigroup. I.e., the operation is commutative and associative over the set D; and for every d_1, d_2, d_3 in D, $d_1 < d_2$ implies $d_1 * d_3 < d_2 * d_3$ and $d_3 * d_1 < d_3 * d_2$.

Once D has been given a recognizable structure, the next step is to attempt to define a function, μ, that maps D into a subset of the real numbers and preserves the essential characteristics of the structure of D. That is, given d_1, d_2, d_3 in D;

(1) $\mu(d_1) = \mu(d_2)$ if and only if $d_1 \sim d_2$

(2) $d_3 = d_1 * d_2$ implies $\mu(d_3) = \mu(d_1) + \mu(d_2)$ assuming that d_1 and d_2 do not intersect.

Many common measurement functions which measure domains with dense order relations can be defined by arbitrarily selecting a member d_0 of D as a unit. (An order relation is dense if, given d_1, d_2 in D such that $d_1 < d_2$, there exists d_3 in D such that $d_1 < d_3 < d_2$. The length, area, volume, and weight measurement functions measure domains with dense order relations while the counting measure does not.) Then any other element d of D is compared with successive multiples of d_0 until a multiple nd_0 is found such that $nd_0 \lesssim d < (n+1)d_0$. ($\lesssim$ means < or $\sim$.) Next an element d_1 is chosen such that $10d_1 \sim d_0$, and a multiple of d_1 is joined to nd_0 such that $nd_0 * n_1d_1 \lesssim d < nd_0 * (n_1+1)d_1$. Similarly, d_2 and n_2 are chosen such that $nd_0 * n_1d_1 * n_2d_2 \lesssim d < nd_0 * n_1d_1 * (n_2+1)d_2$. Continuing in this manner a decimal number $r = n.n_1n_2n_3\ldots$ is built up and used to define the function

$\mu : D \to R^+$, by $\mu(d) = r$.

It should be noted that certain basic assumptions have been made in attributing a structure to D and defining a measurement function from D to R. First and foremost it has been assumed that the attribute that is being measured remains constant under certain transformations and is not affected by the empirical procedures used to define the operation and relations on D. Other assumptions have been made about the reliability and accuracy of these empirical procedures.

The Gal'perin and Georgiev Study

Gal'perin and Georgiev conducted their study at the end of the 1958-1959 school year with 60 Ss from the "upper group" of a Soviet kindergarten. The Ss' ages ranged from 6 yrs., 6 mos. to 7 yrs., 2 mos. They reached the following conclusions based on the given tasks:

Most students do not understand that a

unit may consist of parts (Assignment 2).* Having been shown that two mugs of liquid equal one cup of liquid, the Ss were given three cups and four mugs filled with water and were asked how many cups of water there were. Forty-nine Ss answered incorrectly: 37 counted all the containers individually and 12 counted only the three cups.

A quantity may not be presented as an entity and the units in which it is measured may not be directly identified as entities (Assignment 4). The Ss were given a long cord and asked to measure out pieces equal to four of these cords (a 10-centimeter model was given). Forty Ss answered incorrectly: 11 cut off four 10-centimeter segments, 10 measured the piece of cord arbitrarily, and 19 made some other kind of mistake or were not able to respond at all.

Students are indifferent to the size of the unit of measure (Assignment 5). The Ss were given two identical cups filled with rice and asked to measure the rice in one of the cups into separate piles using a teaspoon and then, using a tablespoon, to measure the rice in the other cup into another group of piles. They were then asked which group of piles contained more rice. Fifty Ss responded that there was more rice in the teaspoon group, which had more piles.

Students are indifferent to the fullness of the unit of measure (Assignment 1). Having been asked to put five spoonfuls of rice on the table and take four away, the Ss were asked how many spoonfuls were left. Thirty-two Ss responded incorrectly because they had not checked the fullness of the spoons. Eighteen indicated there was some number of spoonfuls more than one left, and 10 indicated that there was one spoonful left even though there was a great deal more.

The unit of measure is often regarded not as a tool for isolating units for subsequent counting but as a specific quantity that varies directly as the quantity being measured (Assignment 3). Ss were shown a pile containing ten teaspoons of rice. They were told there were ten spoonfuls of rice in the pile and asked whether the teaspoon or the tablespoon was used to measure the rice in the pile. Twenty-seven Ss answered incorrectly, relating the big size of the pile with the big spoon.

(Assignment 6) The Ss were asked whether there were more spoonfuls in a pile in which they had just measured five teaspoons of rice or in a pile in which he had just measured four tablespoons of rice. Thirty-five Ss answered incorrectly, insisting that there were more spoonfuls in the larger pile.

Many children rely on and have more faith in direct visual comparison of quantities rather than measurement by a given unit of measure (Assignment 7). The Ss were asked whether there was more rice in a pile where they had just placed four spoonfuls of rice (pile I) or in a pile in which they had just placed two spoonfuls of rice which had been spread out by the experimenter (pile II). They were then asked whether there was more rice in pile II or in a third pile which also contained two spoonfuls but which had not been spread out. Twenty-eight Ss chose pile II as the largest of the three. Seventeen recognized that pile I was larger than pile II but did not realize that pile II and pile III contained the same amount of rice.

Gal'perin and Georgiev concluded that the above misconceptions could be attributed to the incorrect characterization of the unit as a discrete entity; therefore, they developed a mathematics program based on a systematic treatment of the relation between units, units of measure, and number. During the 1959-1960 school year, the program was piloted with fifty children from the upper groups of the same kindergartners used in the original investigation. When the test of measurement concepts was administered in the spring, ten of the items were answered correctly by all children, and at most only four children missed any item.

Piaget's Measurement Studies

Whereas Gal'perin and Georgiev concentrated on the measurement function and specifically on the role of the unit in defining the function, Piaget, Inhelder, and Szeminska (1960) have taken a more general view and studied not only how young children assign a number to a quantity but also how well they understand the partition of the domain into equivalence classes and whether they recognize the basic assumption of invariance of the quantity under transformation. For Piaget et al. (1960) the central idea "underlying all measurement is the notion that an object remains constant in size throughout any change in position" (p. 90). This notion of invariance of a property under transformation Piaget calls "conservation."

Based on studies of length, area, and volume, Piaget proposes a stagewise development of measurement, which is interrelated

*The assignment number refers to the number of the task in this study.

with the development of the concept of conservation.

In studying the development of measurement of length, Piaget asked Ss to judge the relative lengths of strips of paper mounted on cardboard sheets in a variety of linear arrangements involving right angles, acute angles, etc. After the Ss had replied, they were given a number of movable strips and asked to verify their judgments. Later they were given short strips of cardboard 3 cm., 6 cm., and 9 cm. long and asked to measure the mounted strips. Similar tasks were assigned in the study of area and volume. Since, however, Piaget found comparable stages of development in these measurements, length only shall be dealt with here.

The earliest stages, I and IIA, Piaget characterized as "a wide variety of responses which have only negative characteristics in common" (Piaget et al, 1960, p. 117). Ss do not conserve length and are totally incapable of using a unit of measure. They generally rely on visual comparisons and have no confidence in measurement. If asked to measure, some Ss simply run the unit along the line, making no subdivisions into equal units. Others only cover part of the line or partition it into unequal sections. No S in this stage realizes the importance of a constant size for the unit of measure.

In substage IIB conservation is dimly perceived, and Ss begin to understand the use of a unit in measuring. By trial and error Ss gradually discover that if it takes more units to cover A than to cover B then A is longer than B. However, they fail to recognize the importance of the size of the unit and often count a fraction of a unit as a whole or equate two lines that measure the same number of units with different size units of measure.

In substage IIIA conservation and the use of a common unit are immediately perceived, but Ss continue to ignore the size and completeness of units of measure. In substage IIIB they recognize the importance of the size of different units of measure and understand the inverse relationship between unit size and number of units. It is at this point that Ss conserve and measure with success.

original task, four teaspoons of rice were poured on the table from a cup. Next to this pile two teaspoons of rice were poured and spread out. The S was asked which pile contained more rice. If he responded correctly, the experiment was completed, as in the original. But if he responded that the spread-out pile contained more, the rice from both piles was measured back into two cups, identical in size and opaque. He was then asked which cup contained more rice. Thus, in this task the S had the same evidence as in the original task 7; however, the order had been inverted so that no confusing physical clues were present when he made his response. This task also provided a measure of how many Ss could respond to task 7 on the basis of which pile looked larger without reference to the number of spoonfuls in each pile.

For task 9, task 5 was repeated as given in the original, except that Ss were asked to measure the rice into opaque cups rather than into piles. Thus, Ss had to base their responses on conservation or on a numerical rather than visual cue as they could do in task 5.

For task 10, task 6 was repeated except that Ss were asked to measure the rice into cups rather than into piles.

In the last task the Ss were shown two glasses containing unequal amounts of rice. The rice was measured into cups using different spoons that were of such a size that each glass measured three spoonfuls. Ss were then asked which glass contained more rice. In this task, as in tasks 8 and 9, Ss were confronted with a numerical clue as to which cup held more rice, a clue that conflicted with their original perceptions of which cup held more rice.

These last three items differed from those in the Soviet study in that the numerical clues followed the visual clues. When Ss decided where there was more rice, they had no visual clues to distract them.

All 11 tasks were individually administered in a room separated from the classroom, where the experimenter and S could be alone with no distractions. Tasks 1 through 7 were administered the first day in sessions that ran about 15 minutes. Tasks 8 through 11 were administered two days later in approximately 10-minute sessions.

All tasks were administered by the experimenter, who was a stranger to the Ss and had not participated in their instruction.

Sample

The sample consisted of twenty first grade Ss randomly selected by the experimenter from five first grade classes. Ten of the Ss were from three first grades in a Madison, Wisconsin, elementary school that was participating in the Developing Mathematical Processes (DMP) mathematics curriculum development project. These students generally came from upper middle class homes in an area where many university professors live.

The other ten students were selected from two classes in an elementary school in a small, prosperous farming community about ten miles from Madison. They studied mathematics from the television-text series Patterns in Arithmetic (PIA).

DMP is an individually guided, activity-learning mathematics program being developed by the staff at the Wisconsin Research and Development Center for Cognitive Learning. Measurement operations are integrated into the program from the beginning and are used as a basis for the development of the natural numbers. By the time this study was administered, Ss had identified length as a property of objects, compared objects on the basis of length, equalized objects on length by adding to the shorter or taking from the longer, and measured objects using an arbitrary unit of measure. They had gone through a similar sequence of comparing, equalizing, and measuring objects on the basis of weight; and some Ss had explored area and volume as properties of objects.

PIA, on the other hand, treats measurement more traditionally. There are two short units on measurement of length. In the first unit Ss had been introduced to the vocabulary of measurement of length, compared objects on the basis of length, measured objects using a given unit of measure, and had been shown that distance is measured along a straight line. In the second lesson they had measured using inches and feet.

When the study was administered, both programs had covered measurement of length, comparison of objects on the basis of length, and the use of units of measurement of length. Neither had specifically covered measurement of volume or the use of teaspoons, tablespoons or cups as units of measure. The only item that dealt with material that was specifically covered in either program was task 4, in which the Ss were asked to measure out a length of

cord; and even this task was somewhat unfamiliar because all the Ss' instruction had been on measuring given lengths, rather than producing a length of a given measure.

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III
Tasks and Results

Individual Tasks Itemized

Task 1. E asked S, "Measure out five spoonfuls [teaspoons] of rice and put them in a pile here. Now take the spoon and put four spoonfuls back in the bowl. How many spoonfuls are left on the table?"

Eighteen Ss missed this item. Twelve indicated there was more than one spoonful left. Five of these said there were more than five spoonfuls. Six Ss responded that there was only one spoonful left even though, because of carelessness in filling the spoons, there was actually much more than one spoonful. None of these six Ss could explain why they could not get all the remaining rice in the spoon.

Task 2. First, by pouring water from a cup (4 oz.) into a mug (8 oz. cup distinguished by handles), S was shown that two full cups made one full mug. He was then shown three mugs and four cups filled with water; and E, indicating the entire collection of cups and mugs with a motion of his hand, asked, "How many mugs of water are there on the table in front of you?"

If S responded "Three," the three mugs of water were removed, and E asked, "How many mugs could I fill with these four cups of water in front of you?"

Eighteen Ss missed this item. Two Ss responded that there were four mugs. One confused the role of mugs and cups and said there were ten mugs. Fifteen Ss just counted the three mugs. Of these fifteen, however, only four did not recognize that the four cups could be used to fill two mugs.

Task 3. A pile of rice was poured on the table from a cup. A teaspoon and a tablespoon were placed on either side of the pile. E said, There are ten spoonfuls of rice in this pile. Which of these spoons was used to measure out the rice?"

This proved to be the easiest item for the American students. Only five Ss missed the item, all of whom said they thought the big spoon was used because it was bigger and would not take so long. Nor did these five Ss imply that they associated the big spoon with the big pile. In fact they seemed rather oblivious to the size of the pile.

Task 4. S was given a long cord and a pair of scissors and asked to "Measure and cut off a piece equal to four of these." A 10 cm. model was given.

Fourteen Ss missed this item. Five Ss carefully measured off four pieces of string equal to the given piece. One S carefully measured off three pieces equal to the given piece, giving him a total of four. Two Ss cut off four unmeasured pieces only approximately equal to the given piece. One S cut off three unmeasured pieces only approximately equal to the given piece. One S cut off one unmeasured piece approximately equal to the given piece. Three Ss cut off a single unmeasured piece about as long as four of the given pieces. One S cut off a piece about as long as four of the given pieces and then cut it in half. Thus of the fourteen Ss who made errors, nine concentrated on getting four pieces, and eight did not measure their pieces. Of the nine Ss who indicated that they did not assimilate the entire set of directions by centering on the four, only two hesitated or asked that the directions be repeated. The other seven proceeded as if they understood exactly what was asked.

Task 5. S was shown two identical glasses with equal amounts of rice in them (two tablespoons). S was asked, "Which glass contains more rice?" If he indicated some difference, this difference was eliminated by pouring more rice into the other glass until the S indicated there was the same amount in each glass. Then the S was given a teaspoon and asked to "mea-

sure the rice in this glass into separate piles on this piece of paper." Then he was given a tablespoon and asked to measure the rice in the other glass onto another piece of paper. E then asked, "Where is there more rice, here or here?"

Eighteen Ss missed this item. Sixteen said there was more rice where there were more piles, and two said there was more rice in the piles measured by the bigger tablespoon. Two of the Ss who said there was more rice in the teaspoon group assumed that two teaspoons made one tablespoon and decided there was more rice in the teaspoon group because there were more than twice as many piles.

Task 6. S was given a teaspoon and asked to "put five spoonfuls of rice in a pile here." He was then given a tablespoon and told to "put four spoonfuls of rice in a pile here." E then asked, "Where are there more spoonfuls of rice, here or here?"

Eleven Ss missed this item, asserting there were more spoonfuls where there was more rice. All eleven remembered where they had put the four and the five spoonfuls.

Task 7. S was given a teaspoon and asked to make three piles of rice—the first containing four spoonfuls, the second containing two spoonfuls, and the third also containing two spoonfuls. E said, "Now watch what I do," and spread out the second pile. E then asked, "Which pile contains more rice?" indicating piles I and II. He then asked, "Which of these two piles contains more rice?" indicating piles II and III.

Fourteen Ss made a mistake on this item. Only six missed the first part, but fourteen insisted there was more rice in the second pile than in the third. Of the eight Ss who answered part one correctly but missed part two, six said there was more in pile II than in pile III because pile II was spread out, but there was more in pile I than pile II because there were four spoonfuls in pile I and only two in pile II. In other words, on the same question these Ss changed the basis for their responses, one time responding on the basis of how the piles looked and one time responding on the number of spoonfuls of rice in the piles.

Task 8. Four teaspoonfuls of rice were poured from a cup in front of the S. Next to this pile two teaspoonfuls of rice were poured and spread out by E. The S was asked, "Where is there more rice, here or here?" If S responded that there was more in the pile containing two spoonfuls, E measured the rice in each pile into different cups and asked, "Which cup contains more rice?" (See task 7.)

Fifteen Ss responded that there was more rice in the pile with four spoonfuls. All five who said that there was more in the spread-out pile said that there was more rice in the can containing four spoonfuls after the rice was measured. None of these five gave any indication that they realized that this answer conflicted with their previous answer.

Task 9. S was shown two identical glasses with equal amounts of rice in them (two tablespoons). S was asked, "Which glass contains more rice?" If S indicated some difference, this difference was eliminated by pouring rice into the cup S thought less full until S indicated there was the same amount of rice in each glass. Then S was given a teaspoon and asked to measure the rice in one of the glasses into a cup. Then he was given a tablespoon and asked to measure the rice in the other glass into another cup. E then asked, "Where is there more rice, in this cup or in this cup?" (See task 5.)

Sixteen Ss missed this item. Nine responded that there was more rice where there were more spoonfuls, and seven answered that there was more rice in the cup that was filled by the tablespoon. Three of the seven associated more rice with the bigger spoon. The other four were rather confused and were not able to articulate the basis for their answer.

Task 10. S was given a teaspoon and asked to "put five spoonfuls of rice in this cup." He was then given a tablespoon and asked to "put four spoonfuls of rice in this cup." E then asked, "Where are there more spoonfuls of rice, in this cup or in this cup?" (See task 6.)

Eight Ss missed this item, asserting there were more spoonfuls in the cup filled by the tablespoon. Several explained this answer by saying that there were more spoonfuls because the spoon was bigger, yet all eight remembered how many spoonfuls they had put in each cup.

Task 11. S was shown two glasses which contained unequal amounts of rice and was asked, "Which glass contains more rice?" E then measured the rice into two cups using different spoons. The spoons looked about the same size, but one had a deeper bowl than the other. Both glasses contained three spoonfuls of rice, though the amount of rice differed because of the different spoon sizes. E then asked, "Which cup contains more rice?"

Only three Ss missed this item. Two said the cups contained equal amounts of rice, and one said the wrong cup contained more rice because that spoon was bigger. Of the seventeen who answered correctly, eight said there was more in the appropriate cup because the

spoon was bigger, seven said there was more because there had been more in the glass, and two were not able to explain their answers.

Correlations Between Tasks

All but one S who did part one of task 7 correctly also did task 8 correctly, though two Ss who missed part one got task 8 right. Thus, although most of the Ss on part one of task 7 responded that pile I was larger than pile II because there were more spoonfuls in pile I, all but one of these Ss could visually distinguish which pile was larger without knowing how many spoonfuls were in either pile.

Table 1
Contingency Table for Tasks 7A and 8

8 \ 7A	-C	C	Total
C	2	13	15
-C	4	1	5
Total	6	14	20

C Correct
-C Not Correct

Task 9, on which Ss measured the rice into cups rather than into separate piles, proved to be slightly easier than task 5, but not much. Both Ss who did task 5 correctly also did task 9 correctly, and two additional Ss did task 9 correctly. The number of spoons emptied into the cups, however, turned out to be practically as strong a distractor as the individual piles, as sixteen Ss still answered incorrectly. On task 9, seven Ss responded that the rice measured by the bigger spoon was the greater quantity, while only two Ss gave this response on task 5.

Table 2
Contingency Table for Tasks 5 and 9

9 \ 5	-C	C	Total
C	2	2	4
-C	16	0	16
Total	18	2	20

C Correct
-C Not Correct

On task 10 the S was no longer distracted by a larger pile in deciding which cup contained more spoonfuls, and this turned out to be considerably easier than task 6. This was to be expected. What is interesting is that, even when no longer confronted with the piles of rice, seven Ss still responded that there were more spoonfuls where they had actually put fewer. Thus, some Ss who did not perform task 6 correctly may not have been responding simply on the basis of the bigger pile. They may have responded on the basis of the larger spoon. Since in task 10 no effort was made to keep the Ss from looking in the cups while they were filling them (though the cups were moved away so they could not be compared after they were both filled), it is also possible that some or all of the Ss remembered how much rice was in each cup and responded on that basis. The fact that two Ss correctly performed task 6 but not task 10 casts doubt on the reliability of this pair of tasks.

The four tasks that involved some degree of conservation fell into a well-defined sequence. All Ss who performed task 5 correctly also got task 9. All Ss who got task 9 also got task 7, and all Ss who got task 7 also got task 11.

Ss' answers to these items also fell into several well-defined categories. Two Ss were correct in all four tasks. Two Ss always responded on the basis of the last stimulus. In other words, they missed all four tasks,

missing both parts of 7. Eleven Ss always found inequality. They missed problems 5, 9, and part two of 7—in which the amounts of rice were really equal—but got 11, where the amounts of rice were unequal. Seven of the eleven got task 7 part one, on which the right or wrong answer entailed choosing the piles to be equal. Thus, these seven were able to con-

Table 3
Contingency Table for Tasks 6 and 10

10 \ 6	~C	C	Total
C	6	7	13
~C	5	2	7
Total	11	9	20

C Correct
~C Not Correct

Table 4
Contingency Table for Tasks 7A and 7B

7B \ 7A	~C	C	Total
C	0	6	6
~C	6	8	14
Total	6	14	20

C Correct
~C Not Correct

Table 5
Contingency Table for Tasks 9 and 11

11 \ 9	~C	C	Total
C	13	4	17
~C	3	0	3
Total	16	4	20

C Correct
~C Not Correct

Table 6
Contingency Table for Tasks 3 and 6

6 \ 3	~C	C	Total
C	2	7	9
~C	3	8	11
Total	5	15	20

C Correct
~C Not Correct

serve quantity and answer correctly as long as the amounts of rice were unequal. In other words, they could conserve as long as they were conserving inequality.

Beyond this there seemed to be very little correlation between tasks, nor were there discernible patterns of responses. Even tasks 3 and 6 were not highly correlated, although they both purportedly measured the lack of knowledge of the inverse relationship between unit size and number of units.

Comparison of Individual School Results

With the small sample size, large variance on the eleven tasks (5.1), and nonequivalent populations, one should be extremely cautious in making any comparison between the two schools in this study. Thus, although one should not attach too much significance to them, there are several differences between the two schools that can be considered.

On three of the four tasks that involved conservation (7, 9, and 11), the students at the Madison elementary school using DMP had at least two more correct answers than the students using PIA. The fourth conservation item (5) was so difficult that only one S at each school answered it correctly.

On task 6, five Ss in the DMP program and four in the PIA program responded correctly; however, when the rice was measured into cups rather than into piles in task 10, the number of correct responses by students in the DMP program remained the same while the number of correct responses by Ss in the PIA program increased 100 percent to eight. This result is confounded by the fact that only three of the Ss in the DMP program responded correctly to both tasks 6 and 10.

The mean number of correct responses on the first seven tasks for Ss in the DMP and PIA programs were 2.4 and 1.8 respectively. The means on the eleven tasks were 5.1 and 4.0 respectively. Neither difference is significant at the .1 level of significance.

Comparison to Soviet Study

As in comparing the two schools in this study, one should not attach too much significance to the differences between Soviet and American students. In addition to all the reasons for caution listed above, there is also the problem of differences in administrative procedures and the fact that Soviet Ss were selected from an "upper group" while the American Ss were from ungrouped classes.

The Soviet students generally did slightly better than the students in this study. They had an average of 2.77 correct answers as opposed to an average of 2.1 correct answers for the American students. Two of the most difficult tasks for the Americans (1 and 2) were two of the easiest for the Soviets. The incorrect responses on task 2 were also markedly different for both groups. Seventy-five percent of the Americans (83 percent of those giving incorrect responses) said that there were three mugs, while only 20 percent of the Soviets (24 percent of those giving incorrect responses) gave this answer.

Task 3, which was far and away the easiest item for the Americans and the only item they performed significantly better than the Soviets, was not significantly easier than many of the other items for the Soviet students.

Table 7
ANOVA—Total Score of First Seven Items
(n = 20)

Source	d.f.	MS	F
Between schools	1	2.45	<1
Within cells	18	2.83	

Table 8
ANOVA—Total Scores of Complete Test
(n = 20)

Source	d.f.	MS	F
Between schools	1	7.2	1.35
Within cells	18	5.32	

Table 9
Number of Correct Solutions in Each Population Group
(In % of total number of subjects)

Group	Task Number 1	2	3	4	5	6	7	8	9	10	11
Soviet totals	47	52	47	47	17	42	25				
DMP	20	0	80	30	10	50	50	90	30	50	100
PIA	0	20	70	30	10	40	10	60	10	80	70
U. S. totals	10	10	75	30	10	45	30	75	20	65	85

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IV
Conclusions

The students in this study generally performed as poorly on Gal'perin and Georgiev's tasks as did their Soviet counterparts. Since the Ss in this study were drawn from a population that is generally above average in academic performance, it is safe to assume that a large majority of American first grade students would have difficulty with this set of tasks.

The fact that Gal'perin and Georgiev's results can be replicated does not mean, however, that their conclusions are valid. Their conclusions are based upon the assumption that first grade students have a definite although erroneous concept of measurement that centers around the unit as a specific entity. They imply that on each task the student was aware that he was supposed to be measuring, and that the operations he performed were his best attempt at measurement. It is highly questionable whether any of these assumptions are justifiable.

In spite of the fact that the word measure was used whenever possible, many of the Ss in this study did not perceive that they should be using a set of measurement operations that follow a well defined set of rules. None of the Ss in the study had any formal instruction in measuring volume and many of the responses were not inconsistent with the way measurement operations are viewed in their general culture. Spoonfuls and glasses are containers to which society attributes rather flexible measuring rules. A "glass of water" is generally not filled to the brim and "spoonfuls" are not usually leveled off to be sure of getting the same amount in each bite. Therefore, it is not surprising that Ss took a casual view of the measurement procedures. This does not mean that they did not understand the basic relationships. They may have understood that variations in measurement procedures would make a difference but not have known what set of procedures they were being asked to apply. This distinction is important in view of the fact that Ss completed the "measurement" operations before they were asked the questions about them and thus were unable to assess the importance of constant unit size, fullness, etc.

Gal'perin and Georgiev concluded that young children are indifferent to the size and fullness of the unit of measure. They implied that young children perceive all units as the same. This was not the case, however, in task 11, where only three Ss equated the two spoons of different size. It was also not the case in task 5, where a number of the Ss commented that it should take fewer spoonfuls to measure the second glass with the tablespoon than had been required to measure the first glass with the teaspoon even though they subsequently missed the item. Furthermore, two Ss chose the group of piles measured by the tablespoon because the spoon was bigger; and in task 9, which was identical to task 5 except that the piles were no longer visible, seven Ss chose the amount of rice measured by the bigger spoon as being more. The fact that so many Ss missed task 5 seems to be due primarily to the dominance of the greater number of piles. A Piagetian explanation, that young children tend to center on a single dominant dimension, appears to be more reasonable than Gal'perin and Georgiev's conclusion that they are totally indifferent to the size of the unit.

Similarly, the only Ss who seemed indifferent to the fullness of the unit of measure in task 1 were the six who erroneously responded that there was only one spoonful left. The other responses could be interpreted to imply that the Ss perceived the pile as containing a variable number of spoonfuls, depending on how full the spoon was filled. They simply did not understand that they were being asked to maintain a constant size. This is a very different from not realizing that variation in the fullness of the unit of measure makes a

difference.

Furthermore, the mistakes on several of the problems seemed to be due as much to the students' not understanding exactly what they were being asked as to any deep-seated misconceptions of measurement concepts. For example, on the second problem, fifteen of the eighteen Ss who missed the problem believed that they were being asked to count the actual mugs. Of the fifteen, eleven knew that the four cups would fill two mugs, but they did not perceive that this fact had anything to do with the question they were being asked.

Similarly, the errors on task 4 seemed to be basically a result of the Ss not understanding what they were asked to do. The difficulties on task 6 also seemed to be due primarily to a matter of interpretation of the question. As a matter of fact, it is not at all clear that the response that Gal'perin and Georgiev identify as correct is right. To arrive at their answer it seems necessary to assume that the units by which the piles of rice were measured out remain as integral parts of the piles, and that the piles cannot be remeasured using a different unit of measure. It would seem natural in comparing the piles to pick a common unit of measure, either the teaspoon or the tablespoon. Several of the Ss brought up this very point in explaining their answers.

From a slightly different point of view, it may be argued that some of the errors involve more than simply a misapplication of measurement concepts. Assuming that the errors in task 1 involve more than a misunderstanding, the difficulties may stem from the children's inability to recognize subset relationships rather than from the belief that the unit is indivisible. (For a discussion of this phenomenon see Flavell, pp. 304–308.)

Thus, although it appears that first-grade students are unable to perform a number of basic measurement operations, it is not clear that these errors are due simply to an inadequate concept of a unit or even to an inability to define a measurement function and assign a number to a quantity. Rather, the most serious errors seem to occur in the most fundamental aspect of the measurement process—that of dealing directly with the domain of elements to be compared on a given attribute which does not change when the elements are transformed. In other words, these errors appear to result from inadequate logical schema for dealing with quantities rather than an incorrect characterization of the unit of measure. Most of the other difficulties not involving conservation concepts could be attributed to inexperience with formal measurement processes or ambiguity of the test items.

The results of this study do indicate, however, that students do not bring to first grade a stable concept of measurement. They have some basic notion that more units of a given size give a greater quantity of the thing being measured, but they do not seem to have a clear idea of the advantages gained by using a constant, accurate unit of measure. Specific instruction on the need for standard units of measure would seem to be a necessary component of instruction on measurement concepts.

It also seems that there is very little transfer of measurement concepts. Teaching students the basic concepts of measurement of length does not mean that they will apply the principles they have learned to the measurement of area, volume, or weight. Students who have learned to use standard units of length will not necessarily measure with standard units of volume, and it appears necessary to re-teach the basic measurement principles with each type of measurement.

There also seems to be very little transfer from one type of task to another. The poor results on task 4 indicate that learning how to measure the number of units in a given quantity is not sufficient to learn to measure out a given number of units from a larger number. Students should be taught measurement operations in a variety of contexts, with great care being taken to be sure that they understand what they are being asked to do.

None of these conclusions are very surprising. There were, however, several very interesting results that are worthy of further consideration.

To date most conservation studies have involved presenting the subject with two equal quantities which are then transformed in such a way that the quantities appear to be unequal. Since the quantities that are used are equal, therefore, a decision that they are unequal is incorrect. Furthermore, representations of the quantities both before and after transformation are physical. That is, the subject observes the quantity; he does not measure it.

The results of tasks 5, 7, 9, and 11 indicate that conservation-type problems involving unequal quantities may be easier than similar problems involving equal quantities. In tasks 9 and 11, only four Ss recognized the importance of different units when the amounts of rice were equal; but seventeen did when the amounts were unequal. The results of task 7—40 percent of the Ss compared the unequal piles on the basis of the number of spoons of rice but compared the piles con-

taining the same number of spoonfuls on the basis of which was spread out the most—provide further evidence that many young children perform better on problems involving unequal quantities and easily shift their basis for making judgments without recognizing any inconsistencies.

Thus, it appears that young children are more likely to conserve and to recognize the importance of certain basic measurement concepts in some types of problems than in others. Problems on which they compare equal quantities seem to be the most difficult and the most likely to produce errors. Some of the difficulties that young children seem to have with conservation and basic measurement concepts may not occur if the child is not asked to compare equal quantities.

The role of numerical stimuli in conservation of quantity also deserves further consideration. The results of tasks 5 and 9 indicate that the number of spoons was almost as strong a distractor as the individual piles. Since the process of assigning a number to a quantity is the basis of measurement, this relationship deserves further study. In fact, the entire relationship between the kind of stimuli, the order of the stimuli, and the role of equality and inequality warrants further study in conservation experiments.

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GPO 827-871-2

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